

# Josephson-like spin current in junctions composed of antiferromagnets and ferromagnets

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We study Josephson-like junctions formed by materials with antiferromagnetic (AF) order parameters. As an antiferromagnet, we consider a two-band material in which a spin density wave (SDW) arises. This could be Fe-based pnictides in the temperature interval  $T_c \leq T \leq T_N$ , where  $T_c$  and  $T_N$  are the critical temperatures for the superconducting and antiferromagnetic transitions, respectively. The spin current  $j_{sp}$  in AF/F/AF junctions with a ballistic ferromagnetic layer and in tunnel AF/I/AF junctions is calculated. It depends on the angle between the magnetization vectors in the AF leads in the same way as the Josephson current depends on the phase difference of the superconducting order parameters in S/I/S tunnel junctions. It turns out that in AF/F/AF junctions, two components of the SDW order parameter are induced in the F-layer. One of them oscillates in space with a short period  $\xi_{Fb} \sim \hbar v / \mathcal{H}$  while the other decays monotonously from the interfaces over a long distance of the order  $\xi_{N,b} = \hbar v / 2\pi T$  (where  $v$ ,  $\mathcal{H}$  and  $T$  are the Fermi velocity, the exchange energy and the temperature, respectively; the subindex b denotes the ballistic case). This is a clear analogy with the case of Josephson S/F/S junctions with a nonhomogeneous magnetization where short- and long-range condensate components are induced in the F-layer. However, in contrast to the charge Josephson current in S/F/S junctions, the spin current in AF/F/AF junctions is not constant in space, but oscillates in the ballistic F-layer. We also calculate the dependence of  $j_{sp}$  on the deviation from the ideal nesting in the AF/I/AF junctions. The spin current is maximal in the insulating phase of the AF and decreases in the metallic phase. It turns to zero at the Neel point when the amplitude of the SDW is zero and changes sign for certain values of the detuning parameter.

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## I. INTRODUCTION

Similarity between the magnetic and superconducting ordering has been noted long time ago (see Ref. 1 and the review articles 2 and 3). In the simplest case of the BCS superconductivity the order parameter is described by a complex quantity  $\Delta = |\Delta| \exp(i\chi)$  with the amplitude  $|\Delta|$  that determines the Cooper pair density and the phase  $\chi$  related to a voltage  $V$  via the gauge Josephson equation

$$\hbar \partial_t \chi = qV, \quad (1)$$

with  $q = -2|e|$ ,  $e$  being the elementary charge. This relation shows that, in equilibrium, the gauge-invariant quantity  $\mu$  equals zero:  $\mu = \hbar \partial_t \chi - 2eV$ .

The electrical current  $\mathbf{j}_q$  of the condensate is expressed in terms of another gauge-invariant quantity—the condensate velocity  $\mathbf{v}_s = (\hbar \nabla \chi - q\mathbf{A}/c)/2m$  as  $\mathbf{j}_q = qn_s \mathbf{v}_s$ .

One of the most remarkable phenomena arising in superconducting systems is the Josephson effect. Josephson<sup>15</sup> showed that a condensate current

$$j_j = j_c \sin(\varphi) \quad (2)$$

can flow in an S/I/S junction being a periodic function of the gauge-invariant phase difference between superconductors  $\varphi = -(2m/\hbar) \int_L^R \mathbf{v}_s \cdot d\mathbf{l}$ , where the limits of the integration  $R$  and  $L$  mean respectively the right and left superconductors  $S$  separated by an insulating layer  $I$ , and  $j_c$  is the Josephson critical current.

In the case of a magnetic ordering—ferromagnetic (F) or antiferromagnetic (AF)—the order parameter is the magne-

tization  $\mathbf{M}$  (in the antiferromagnets the vector  $\mathbf{M}$  is the magnetization in one of the sublattices). The absolute value of  $\mathbf{M}$  corresponds to the amplitude  $|\Delta|$  in superconductors, and the angle  $\theta$  between the  $z$ -axis and the vector  $\mathbf{M}$  is analogous, to some extent, to the phase  $\chi$ . The temporal variation of  $\theta$  is described by the equation similar<sup>5,6</sup> to Eq. (1),

$$\partial_t \theta = -g\mu_B \delta \mathbf{B} \cdot (\mathbf{M}_R \times \mathbf{M}_L) / M_R M_L, \quad (3)$$

where  $g$  is the gyromagnetic ratio,  $\mu_B$  is the Bohr magneton,  $\delta \mathbf{B} = \mathbf{B}_R - \mathbf{B}_L$  and  $\mathbf{B}_{R,L}$  is the magnetic induction in the respectively right or left F- or AF-layer composing the Josephson-like structure. In Eq. (3), the  $z$ -axis is chosen to be directed along the vector  $\delta \mathbf{B}$ .<sup>7</sup>

The Josephson-like effect in F/I/F and AF/I/AF junctions was studied in Refs. 4–6. It was found that in these junctions, the spin current has the form

$$j_{sp} = j_{c,sp} \sin(\alpha), \quad (4)$$

where  $\alpha = \theta_R - \theta_L$  and  $j_{c,sp}$  is a constant related to the barrier transmittance  $|T|^2$ .

In the full analogy to the Josephson charge current  $j_j$ , the spin current is nondissipative. The absence of the dissipation for the spin current and the possibility to use this dissipationless current in spintronics has been discussed in many papers<sup>7–11</sup> (see also reviews 12 and 13). Unfortunately, using the spin currents in spintronics does not seem to be straightforward because the analogy between the spin and charge currents is not complete, namely, the spin current is not a conserved quantity.<sup>1–3</sup> We will demonstrate the validity of this statement considering the proximity effect in

AF/F structures. In particular, we will calculate the spin current through a Josephson-like AF/F/AF junction.

It is well known that the Josephson effect in S/F/S junctions reveals new remarkable properties<sup>16–19</sup> that are absent in the case of more conventional S/I/S or S/N/S junctions.

First, the condensate functions  $f$  penetrating into a diffusive F-layer decay on a very short range of the order  $\xi_{Fd} = \sqrt{D\hbar/\mathcal{H}}$ , where  $D$  is the classical diffusion coefficient of the ferromagnet and  $\mathcal{H}$  is the exchange field; the subindex  $d$  denotes the diffusive case. This expression for  $\xi_{Fd}$  is valid if the condition  $\tau\mathcal{H} \ll 1$  is fulfilled, where  $\tau$  is the momentum relaxation time. Usually, the exchange field  $\mathcal{H}$  is much larger than the critical temperature of the superconducting transition  $T_c$ . In addition, the Cooper pairs wave function oscillates in space. It consists of the singlet component  $f_3$ , and the triplet component  $f_0$  with zero projection of the total spin on the direction of the magnetization  $\mathbf{M}$ . Such oscillations lead to a change of the sign of the Josephson critical current  $j_c$ , and the so called  $\pi$ -state is realized in the junction.

Another interesting effect predicted in Ref. 20 (see also the reviews 18 and 19) and observed in several experiments<sup>21–28</sup> in S/F structures, is related to a new type of superconducting correlations. This type of correlations called “odd triplet superconductivity” can arise due to the proximity effect in the F-layer with a nonhomogeneous magnetization. In the case of a uniform magnetization in the F-layer, the pair wave function there has both the singlet component  $f_{\text{sng}} \sim \langle \psi_\uparrow(t)\psi_\downarrow(0) - \psi_\downarrow(0)\psi_\uparrow(t) \rangle$  and the triplet component  $f_0 \sim \langle \psi_\uparrow(t)\psi_\uparrow(0) + \psi_\downarrow(0)\psi_\downarrow(t) \rangle$  with zero projection of the total spin of the Cooper pair on the direction of the magnetization. Both components oscillate in space and decay on a short length of the order of  $\xi_{Fd}$ . In the ballistic case, these components oscillate with the period  $\sim \xi_{Fb} = v\hbar/2\mathcal{H}$ .

The situation changes drastically if the magnetization in F is not uniform in the nearest vicinity of the S/F interface.<sup>18–20</sup> In this case, not only the components  $f_{\text{sng}} \equiv f_3$  and  $f_0$  arise in the F-layer, but also the triplet component  $f_1(t) \sim \langle \psi_\uparrow(t)\psi_\uparrow(0) + \psi_\uparrow(0)\psi_\uparrow(t) \rangle$  with nonzero projection of the total spin on  $\mathbf{M}$  is generated. This component is insensitive to the exchange field and penetrates the F-layer over a long (compared to  $\xi_F$ ) distance. The penetration length does not depend on the exchange field  $\mathcal{H}$  and is much larger than  $\xi_F$ . In absence of the spin-orbit interaction and scattering by magnetic impurities this length may become comparable with the penetration length of the superconducting condensate into the diffusive normal (nonmagnetic) metal  $\xi_{Nd} = (D\hbar/2\pi T)^{1/2}$ .

The component  $f_1$  can be called the long-range triplet component (LRTC). It is symmetric in the momentum space and is therefore also insensitive to the scattering by ordinary (nonmagnetic) impurities. In this respect the LRTC differs drastically from the usual triplet component  $f_\text{tr}$  that describes superconductivity in  $\text{Sr}_2\text{RuO}_4$ .<sup>29</sup> The latter is an antisymmetric function of the momentum  $\mathbf{p}$  and is therefore destroyed by ordinary impurities. The correlator  $f_1$  changes sign by permutation of the operators  $\psi_\uparrow(t)$

and  $\psi_\uparrow(0)$ , as it should be according to the Pauli principle, which follows from the fact that the Fermi operators  $\psi_\uparrow(t)$  and  $\psi_\uparrow(0)$  anticommute at equal times. This means that  $f_1(0) \sim \langle \psi_\uparrow(0)\psi_\uparrow(0) + \psi_\uparrow(0)\psi_\uparrow(0) \rangle = 0$ , i.e., the function  $f_1(t)$  is an odd function of time  $t$  or, in other words, it is an odd function of the frequency  $\omega$  in the Fourier representation. That is why this component is called “odd triplet component”.

Recent experiments unambiguously confirmed the existence of the LRTC in the S/F structures with a nonhomogeneous magnetization.<sup>21–28</sup>

In this paper we show that the analogy between the superconducting and magnetic order parameters in Josephson-like junctions is deeper than it was indicated previously. It turns out that in the SDW/F/SDW junction with a ballistic F-layer the “short”- and long-range AF order parameters are induced in the F-layer in an AF/F structure (we use the quotation marks for “short”-range, to indicate that in the considered ballistic case this component does not decay on a short range, but rather oscillates with a short period). The component of the AF order parameter with the  $\mathbf{M}$  vector (in each sublattice) being parallel to the magnetization vector in the F-layer  $\mathbf{M}_F$ , which is supposed to be oriented along the  $z$ -axis, penetrates into the F-layer and oscillates with a short period of the order of  $\xi_{Fb}$ . On the other hand, the SDW with the  $\mathbf{M}$  vector perpendicular to  $\mathbf{M}_F$  penetrates the F-layer over a long distance of the order of  $\xi_{Nb}$  and decays monotonously. This part of the SDW order parameter may be called the long-range component induced in the ferromagnetic layer.

At the same time, in spite of the long-range penetration of the AF correlations into the ferromagnet F the spin current that arises at the AF/F interface is not constant in space. It oscillates in the F region with a short period. The study of the spin current in various heterostructures is important not only from the point of view of fundamental physics but also from the point of view of possible applications of these structures in spintronics.<sup>13</sup>

We do not analyze the case of the Bose–Einstein condensation in the magnon system at a high magnon density which can be realized via the external pumping. In this case, a Josephson-like effect may also arise, but it has different properties (see the review 14 and references therein).

## II. MODEL

In order to describe the AF order we adopt the model developed for superconducting Fe-based pnictides.<sup>31–35</sup> In this model the band structure is assumed to consist of two bands (electron and hole) with perfect or almost perfect nesting, which is taken into account by a certain parameter  $\delta\mu$ . Due to this, a spin density wave (SDW) corresponding to an antiferromagnetic order parameter arises in these materials. On the basis of this model, which describes well many properties of this new class of high- $T_c$  superconductors (see review articles 36–41 and references therein), the generalized Eilenberger equations for the quasiclassi-

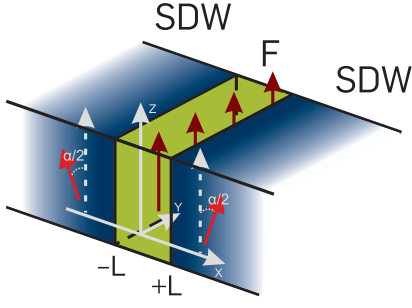


FIG. 1. (Color online.) The considered setup. The two antiferromagnets SDW are separated by a thin ferromagnetic layer of thickness  $2L$ . The orientation of the magnetization vectors of the SDW in the leads lying in the  $y$ - $z$ -plane is shown in red.

cal Green's functions  $\check{g}$  have been derived in the previous paper<sup>42</sup> by the authors (see also a recent paper<sup>52</sup> where the definition of the Green's functions differs from ours). The quasiclassical formalism is especially suitable for tackling nonhomogeneous problems and the matrix functions  $\check{g}$  allow one to find all the necessary observable quantities, such as the charge and spin currents, the density-of-states, etc. The technique of quasiclassical Green's functions is presented, e.g., in Refs. 43–45.

### III. SDW/F/SDW “JOSEPHSON” JUNCTION

#### A. “Short”- and Long-range Components

First, we consider a Josephson-like AF/F/AF junction composed of two leads and a thin ferromagnetic layer between the leads. In the leads a spin density wave exists, which is a particular example of the AF order and the layer between these materials is a ferromagnet in the ballistic regime (see Fig. 1). For simplicity, we assume that both the SDW and the F parts of the structure consist of similar two-band materials and differ from each other by different interaction constants and doping levels. The AF/F interface penetrability is supposed to be small and thus one can neglect the effect of the F-layer on the SDW leads. The Green's functions in the right (left) leads are given by the formula that can be obtained from Eq. (6.4) of Ref. 42 by setting the superconducting order parameter to zero,  $\Delta = 0$ )

$$\check{g}_{R(L)} = \frac{1}{\mathcal{E}_M} \{ \omega_n \hat{\tau}_3 + W_{M0} [ \hat{\rho}_1 \cdot \hat{\tau}_2 \cdot \hat{\sigma}_3 \cos(\alpha/2) \pm \hat{\rho}_2 \cdot \hat{\tau}_1 \cdot \hat{\sigma}_2 \sin(\alpha/2) ] \}. \quad (5)$$

Here,  $\omega_n = \pi T(2n+1)$  is the Matsubara frequency,  $W_{M0}$  is the amplitude of the SDW,  $\mathcal{E}_M = \sqrt{\omega_n^2 + W_{M0}^2}$  and  $\hat{\rho}_i \cdot \hat{\tau}_j \cdot \hat{\sigma}_k$  is the tensor product of Pauli matrices in the band, Gor'kov–Nambu and spin space, respectively. The magnetization of the SDW lies in the  $y$ - $z$ -plane at the angle  $\pm\alpha/2$  relative to the  $z$ -axis, with the projection on the  $y$ -axis  $\pm W_{M0} \sin(\alpha/2)$  in the left (right) leads. For simplicity, we suppose the ideal

nesting and set  $\delta\mu = 0$ . In the F-layer, the quasiclassical Green's functions obey the equation

$$n_x v \partial_x \check{g} + [\check{\Lambda}, \check{g}] = 0, \quad (6)$$

where  $n_x = p_x/p$  and  $v$  is the modulus of the Fermi velocity. The matrix  $\check{\Lambda}$  is defined as  $\check{\Lambda} = \hat{\tau}_3 \cdot (\omega_n \hat{\rho}_0 \cdot \hat{\sigma}_0 + i\mathcal{H} \hat{\rho}_0 \cdot \hat{\sigma}_3)$  with the exchange field in the ferromagnet  $\mathcal{H}$ . We assume that there is no impurity scattering in the F-layer (ballistic case). The impurity scattering leads to a suppression of the SDW amplitude.<sup>51,52</sup>

Equation (6) is supplemented by the boundary condition which we write in the form

$$(\check{g}(n_x) - \check{g}(-n_x)) = \pm \text{sgn}(n_x) [\check{g}, \check{\mathcal{T}} \cdot \check{g}_{R(L)} \cdot \check{\mathcal{T}}]. \quad (7)$$

The transmission coefficient  $\check{\mathcal{T}}$  is a matrix, which is assumed to be small. This boundary condition generalizes the widely used boundary condition derived by Zaitsev<sup>46</sup> to the case of a two-band metal with an SDW. We consider a simplified model for the AF/F interface and assume that there are no interband transitions. In this case, the matrix  $\check{\mathcal{T}}$  has the form  $\check{\mathcal{T}} = (\mathcal{T}_3 \hat{\rho}_3 + \mathcal{T}_0 \hat{\rho}_0) \cdot \hat{\tau}_0 \cdot \hat{\sigma}_0$ , where  $\mathcal{T}_{0,3} = (T_1 \mp T_2)/2$  and  $T_{1,2}$  are the matrix tunneling elements for transitions from the band 1 resp. 2 of the F-layer to the same bands 1 resp. 2 of the SDW leads (see the Appendix A). If these elements are equal, i.e.,  $T_1 = T_2 \equiv T_{\text{tun}}$ , then  $\mathcal{T}_0 = 0$  and  $\mathcal{T}_3 = T_{\text{tun}}$ .

The boundary conditions that describe real materials are more complicated. They must include the transitions between different bands and this process may be accompanied by spin flips. However, we are not interested in exact calculations of the amplitude of the SDW penetrating into the ferromagnetic layer, but rather in a qualitative effect—the appearance of “short”- and long-range components. The existence of these components does not depend on the exact form of the boundary conditions.

In the lowest order of approximation in the transmission coefficient  $\mathcal{T}$  (no proximity effect), the matrix  $\check{g}$  in the ferromagnet has the form  $\check{g} \equiv \check{g}_{0F} = \text{sgn}(\omega) \hat{\rho}_0 \cdot \hat{\tau}_3 \cdot \hat{\sigma}_0$ . Therefore, the boundary condition can be written in the form

$$(\check{g}(n_x) - \check{g}(-n_x)) = \pm \text{sgn}(n_x) \mathcal{T}_{\text{eff}}^2 [\hat{\tau}_3, \check{f}_{R(L)}], \quad (8)$$

where the matrix  $\check{f}_{R(L)}$  equals

$$\check{f}_{R(L)} = \frac{W_{M0}}{\mathcal{E}_M} \{ \hat{\rho}_1 \cdot \hat{\tau}_2 \cdot \hat{\sigma}_3 \cos(\alpha/2) \pm \hat{\rho}_2 \cdot \hat{\tau}_1 \cdot \hat{\sigma}_2 \sin(\alpha/2) \} \quad (9)$$

and

$$\mathcal{T}_{\text{eff}}^2 = -(\mathcal{T}_0^2 - \mathcal{T}_3^2) = T_1 T_2 \quad (10)$$

is an effective transmission coefficient for the SDW. The boundary condition (8) defines the spin current through the interface. One can see that if one of the tunneling elements ( $T_1$  or  $T_2$ ) is zero, then the spin current turns to zero.

We need to find a solution of Eq. (6) supplemented with the boundary condition (8). In the considered case of a low penetrability of the SDW/F interface, the matrix  $\check{g}$

in the F-film can be represented in the form  $\check{g} = \check{g}_{0F} + \delta\check{g}$ , where  $\delta\check{g}$  is a matrix with a small “amplitude” which describes the SDW induced in the F-film due to the proximity effect. As usual, we represent  $\check{g}$  as a sum of a symmetric in momentum part and the antisymmetric one, i.e.,  $\delta\check{g} = \check{s} + n_x \check{a}$ . We substitute this expression into Eq. (6) and split it into two equations for the symmetric and antisymmetric parts

$$v\partial_x \check{s} + [\check{\Lambda}, \check{a}] = 0, \quad (11)$$

$$n_x^2 v\partial_x \check{a} + [\check{\Lambda}, \check{s}] = 0. \quad (12)$$

Excluding the matrix  $\check{a}$ , we obtain one equation for the matrix  $\check{s}$

$$-v^2 n_x^2 \partial_x^2 \check{s} + [\check{\Lambda}, [\check{\Lambda}, \check{s}]] = 0. \quad (13)$$

For the matrix  $\check{a}$  we obtain a similar equation. The commutator  $[\check{\Lambda}, [\check{\Lambda}, \check{s}]]$  is easily calculated resulting in

$$[\check{\Lambda}, [\check{\Lambda}, \check{s}]] = 2\{(\omega^2 - \mathcal{H}^2)\check{s} + 2i\mathcal{H}\omega(\hat{\sigma}_3 \cdot \check{s} + \check{s} \cdot \hat{\sigma}_3) + \omega^2 \check{s} - \mathcal{H}^2 \hat{\sigma}_3 \cdot \check{s} \cdot \hat{\sigma}_3\}. \quad (14)$$

Here, we used the property  $\check{s} \sim \check{t}_{1,2}$ . It is seen from the boundary conditions (8–9), that this matrix anticommutes with  $\check{t}_3$ . It is convenient to represent the matrix  $\check{s}$  as a sum of two matrices  $\check{s} = \check{s}_{\parallel} + \check{s}_{\perp}$ , where  $\check{s}_{\parallel} = \hat{s}_0 \cdot \hat{\sigma}_0 + \hat{s}_3 \cdot \hat{\sigma}_3$  and  $\check{s}_{\perp} = \hat{s}_2 \cdot \hat{\sigma}_2$ . We will see that the matrix  $\check{s}_{\parallel}$  describes a “short”-range component of the SDW penetrating into the F-layer, while  $\check{s}_{\perp}$  determines the long-range component, which is perpendicular to the orientation of the magnetization in the ferromagnet. From Eq. (13) we obtain the equations for  $\check{s}_{\parallel}$  and  $\check{s}_{\perp}$

$$-v^2 n_x^2 \partial_x^2 \check{s}_{\parallel} + 4(\omega + i\mathcal{H}\hat{\sigma}_3)^2 \check{s}_{\parallel} = 0, \quad (15)$$

$$-v^2 n_x^2 \partial_x^2 \check{s}_{\perp} + 4\omega^2 \check{s}_{\perp} = 0. \quad (16)$$

In the first order in the transmission coefficient  $\mathcal{T}_{\text{eff}}^2$  the solutions of these equations obeying the boundary conditions are

$$\check{s}_{\parallel} = 2ms_{\omega}\mathcal{T}_{\text{eff}}^2 \hat{\rho}_1 \cdot \hat{t}_2 \cdot [\hat{\sigma}_3 \cdot \mathcal{R}(x) + i\hat{\sigma}_0 \cdot \mathcal{I}(x)] \cdot \cos(\alpha/2), \quad (17)$$

$$\check{s}_{\perp} = 2ms_{\omega}\mathcal{T}_{\text{eff}}^2 \hat{\rho}_2 \cdot \hat{t}_1 \cdot \hat{\sigma}_2 \cdot \mathcal{C}(x) \cdot \sin(\alpha/2), \quad (18)$$

where  $s_{\omega} \equiv \text{sgn}(\omega)$  and  $m = W_{M0}/\mathcal{E}_M$  is the dimensionless amplitude of the SDW induced in the F-layer and

$$\mathcal{R}(x) = \Re\{\mathcal{I}(x)\}, \quad \mathcal{I}(x) = \Im\{\mathcal{I}(x)\}, \quad (19)$$

and

$$\mathcal{I}(x) = \frac{\cosh(\theta_+ x/L)}{\sinh(\theta_+)}, \quad \mathcal{C}(x) = \frac{\sinh(\theta_{\omega} x/L)}{\cosh(\theta_{\omega})}, \quad (20)$$

with  $\theta_{\omega} \equiv \kappa_{\omega} L$  and  $\theta_+ = \kappa_+ L$ . The here introduced wave vectors  $\kappa_+ = 2(|\omega| + i\mathcal{H}\text{sgn}(\omega))/v|n_x|$  and  $\kappa_{\omega} = 2|\omega|/v|n_x|$  determine the characteristic lengths which describe the penetration of the “short”- and long-range components of the

SDW into the F-region, respectively. The same wave vectors characterize the penetration of the short-range component, i.e., the singlet and triplet component with zero projection of the total spin onto the z-axis, and the long-range triplet component of the superconducting wave function into the F-region of an S/F bilayer in the ballistic case.<sup>17–19</sup>

The component of the SDW  $\check{s}_{\parallel}$ , Eq. (15), which can be called the “short”-range component, oscillates at a given angle  $\chi = \arccos(n_x)$  rather fast in space with a period of the order of  $\xi_{\text{Eb}}$  (it is assumed that  $\mathcal{H} \gg T$ ). Being averaged over the angle, it decays from the interface in a power-law fashion. The long-range component  $\check{s}_{\perp}$ , Eq. (16), penetrates into the ferromagnet over a long length  $\sim \xi_{\text{Nb}}$ , which does not depend on the exchange field  $\mathcal{H}$ . Thus, there is an analogy with an S/F/S Josephson junction where the ferromagnet has an inhomogeneous magnetization.<sup>18,19</sup> In the ballistic case, the “short”-range component of the condensate wave function oscillates in space with a period  $\sim \xi_{\text{Eb}}$ ,<sup>17–19</sup> whereas the long-range component penetrates the ferromagnet over the length  $\sim \xi_{\text{Nb}}$ . We use quotation marks for the “short”-range component because, in the considered ballistic case, this component does not decay exponentially in the ferromagnetic layer, but oscillates and decays in a power-law fashion after averaging over the angles. In the diffusive limit in the S/F/S junctions, this component decays exponentially on a short distance of the order  $\xi_{\text{Fd}}$ .

Note that the “short”-range component  $\check{s}_{\parallel}$  consists of the odd and even in  $\omega$  parts (correspondingly, the first and the second terms in Eq. (17)), whereas the long-range component  $\check{s}_{\perp}$  is an odd function of the frequency  $\omega$ . The same dependence has an anomalous (Gor’kov’s) Green’s function in an S/F/S system with a nonhomogeneous magnetization  $\mathbf{M}$ .<sup>17,18</sup> In the last case, the short-range part consists of the singlet component and triplet component with zero projection of the total spin onto the  $\mathbf{M}$  vector, and the long-range part consists of the triplet component with a non-zero projection of the total spin onto the  $\mathbf{M}$  vector.

In Fig. 2 a) we plot the coordinate dependence of the “short”- (the functions  $\mathcal{R}(x)$  and  $\mathcal{I}(x)$ ) and the long-range (the function  $\mathcal{C}(x)$ ) components. The former stems from the part of the matrix  $\check{g}$  corresponding to the magnetization component of the SDW  $\mathbf{M}$  parallel to the magnetization in the ferromagnet  $\mathbf{M}_F||\mathbf{z}$ , i.e., the second term on the right in Eq. (5). The long-range component stems from the SDW component perpendicular to the  $\mathbf{M}_F$  vector and turns to zero at  $\alpha = 0$ .

The components of  $\check{a}$  that determine the spin current  $j_{\text{Sp}}^{x(y)}(x)$  can be presented in the same form as the  $\check{s}_{\parallel,\perp}$  matrix, i.e.,  $\check{a} = \check{a}_{\parallel} + \check{a}_{\perp}$ . The “longitudinal”— $\check{a}_{\parallel}$ , and the “transverse”— $\check{a}_{\perp}$ , parts are related with  $\check{s}_{\parallel,\perp}$  through the simple formulas (see Eq. (12))

$$vn_x^2 \partial_x \check{a}_{\parallel} = 2s_{\omega} \hat{t}_3 \cdot (|\omega| \hat{\sigma}_0 + i\mathcal{H} \hat{\sigma}_1) \cdot \check{s}_{\parallel}, \quad (21)$$

$$vn_x^2 \partial_x \check{a}_{\perp} = 2s_{\omega} |\omega| \hat{t}_3 \cdot \hat{\sigma}_0 \cdot \check{s}_{\perp}. \quad (22)$$



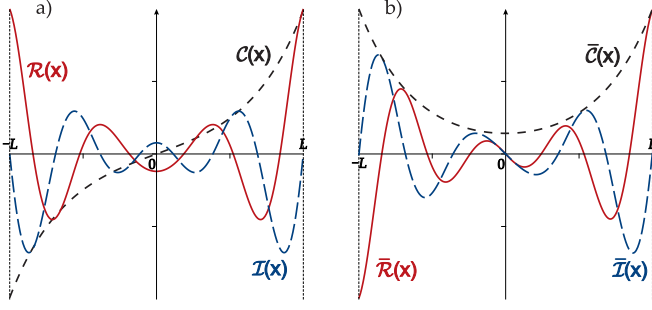


FIG. 2. (Color online.) The coordinate dependence of the “short”-range resp. the long-range components of the Green's function in the F-layer.

a) the symmetric in momentum part of  $\check{g}$ — $\mathcal{R}(x)$  and  $\mathcal{I}(x)$  resp.  $\mathcal{C}(x)$   
 b) the antisymmetric in momentum part of  $\check{g}$ — $\bar{\mathcal{R}}(x)$  and  $\bar{\mathcal{I}}(x)$  resp.  $\bar{\mathcal{C}}(x)$

From Eqs. (15–22) we find the matrix  $\check{a}$ :

$$\hat{\tau}_3 \cdot \check{a}_{\parallel} = -2m \frac{\mathcal{T}_{\text{eff}}^2}{|n_x|} \hat{\rho}_1 \cdot \hat{\tau}_2 \cdot [\hat{\sigma}_3 \cdot \bar{\mathcal{R}}(x) + i\hat{\sigma}_0 \cdot \bar{\mathcal{I}}(x)] \cdot \cos(\alpha/2), \quad (23)$$

$$\hat{\tau}_3 \cdot \check{a}_{\perp} = -2m \frac{\mathcal{T}_{\text{eff}}^2}{|n_x|} \hat{\rho}_2 \cdot \hat{\tau}_1 \cdot \hat{\sigma}_2 \cdot \bar{\mathcal{C}}(x) \cdot \sin(\alpha/2), \quad (24)$$

where

$$\bar{\mathcal{R}}(x) = \Re\{\bar{J}(x)\}, \quad \bar{\mathcal{I}}(x) = \Im\{\bar{J}(x)\}, \quad (25)$$

and

$$\bar{J}(x) = \frac{\sinh(\theta_+ x/L)}{\sinh(\theta_+)}, \quad \bar{\mathcal{C}}(x) = \frac{\cosh(\theta_\omega x/L)}{\cosh(\theta_\omega)}. \quad (26)$$

These functions are plotted in Fig. 2 b) and contribute to the “short”- and the long-range components of the Green's function in the F-layer, more exactly—to the odd in  $n_x$  part of it.

As it follows from the expression for the spin current (see Eq. (3.13) in Ref. 42), the obtained formulas for  $\check{a}_{\parallel,\perp}$  lead to zero spin current inside the F-film. This result is quite natural because the spin Josephson-like current in SDW/F/SDW junction should be proportional to the penetration probabilities of both interfaces, i.e.,  $j_{\text{Sp}}^{x(y)}(x) \sim \mathcal{T}_{\text{eff(L)}}^2 \mathcal{T}_{\text{eff(R)}}^2$ . In the symmetric case under consideration  $\mathcal{T}_{\text{eff(L)}}^2 = \mathcal{T}_{\text{eff(R)}}^2 = \mathcal{T}_{\text{eff}}^2$ . In order to obtain a finite spin current, we have to calculate the matrix  $\check{a}$  in the next order in the transmission coefficient  $\mathcal{T}_{1,2}^2$ . We can easily find the corrections  $\delta\check{a}_{\parallel,\perp}$  from the normalization condition

$$\check{g} \cdot \check{g} = 1. \quad (27)$$

As it follows from this equation

$$\check{a}_t \cdot (\check{g}_{0F} + \check{s}_t) + (\check{g}_{0F} + \check{s}_t) \cdot \check{a}_t = 0, \quad (28)$$

where  $\check{a}_t = \check{a} + \delta\check{a}$  and  $\check{s}_t = \check{s} + \delta\check{s}$ .

Therefore, for  $\delta\check{a}$  we obtain the equation

$$2\check{g}_{0F} \cdot \delta\check{a} + \check{a} \cdot \check{s} + \check{s} \cdot \check{a} = 0, \quad (29)$$

where  $\check{g}_{0F} = \text{sgn}(\omega) \hat{\rho}_0 \cdot \hat{\tau}_3 \cdot \hat{\sigma}_0$ . We are interested in the component  $\delta\check{a}_3$  of the matrix  $\delta\check{a}$ , which is proportional to the matrix  $\hat{\tau}_3$ , because only this component contributes to the spin current. This component commutes with the matrix  $\check{g}_{0F}$ .

Substituting expressions (17–26) into Eq. (29), we obtain for  $\delta\check{a}_3$

$$\delta\check{a}_3 = 2is_\omega \frac{(m\mathcal{T}_{\text{eff}}^2)^2}{|n_x|} \Re\left\{ \frac{\cosh[(\theta_+ - \theta_\omega)x/L]}{\sinh(\theta_+) \cosh(\theta_\omega)} \right\} \sin(\alpha) \hat{\rho}_3 \cdot \hat{\tau}_3 \cdot \hat{\sigma}_1. \quad (30)$$

One can write Eq. (30) in the form

$$\delta\check{a}_3 = 2is_\omega \frac{(m\mathcal{T}_{\text{eff}}^2)^2}{|n_x|} \tanh(\theta_\omega) \frac{\cos(\theta_{\mathcal{H}})}{\cosh^2(\theta_\omega) - \cos^2(\theta_{\mathcal{H}})} \times \cos(\theta_{\mathcal{H}} x/L) \sin(\alpha) \hat{\rho}_3 \cdot \hat{\tau}_3 \cdot \hat{\sigma}_1, \quad (31)$$

where  $\theta_{\mathcal{H}} = \mathcal{H}L/v|n_x|$ .

Now, the spin current can be easily calculated.

## B. Spin Current

We see that the SDW/F/SDW junctions are very similar to the S/F/S Josephson junctions. In analogy to the “short”- and long-range components of the superconducting condensate penetrating the ferromagnet, there are similar “short”- and long-range components of the SDW penetrating the ferromagnet.

Nevertheless, there is an essential difference between the two systems. The Josephson charge current  $j_j$  in an S/F/S junction is constant in space in the ferromagnet. It is determined by the long-range component, provided the magnetization  $\mathbf{M}(x)$  in the F-layer is not uniform and the thickness  $d_F$  of this layer is not too small ( $d_F \gg \xi_F$ ).

In contrast, the spin current in an SDW/F/SDW junction is not constant in the F-layer but oscillates with a period of the order of  $\xi_F$  (see Fig. 3). It is constant in space only in the absence of the exchange field, i.e., in SDW/N/SDW junction.

The spin current through the SDW/F/SDW junction can easily be calculated using the formula for  $j_{\text{Sp}}$  (see Eq. (3.13) of Ref. 42)

$$j_{\text{Sp}}^{x(y)}(x) = -i \frac{(2\pi T) v v}{8} \mu_B \sum_{\omega} \text{Tr} \langle (\hat{\rho}_3 \cdot \hat{\tau}_3 \cdot \hat{\sigma}_{1,2} \cdot \check{a}) n_x^2 \rangle. \quad (32)$$

The notation  $j_{\text{Sp}}^{x(y)}(x)$  means the spin current density with the spin projection  $x$  (resp.  $y$ ) flowing perpendicular to the S/F interface and the angle brackets denote the angle averaging.

Substituting Eq. (30) into Eq. (32) we obtain

$$j_{\text{Sp}}^x(x) = j_{\text{c,Sp}}^x(x) \cdot \sin(\alpha), \quad (33)$$

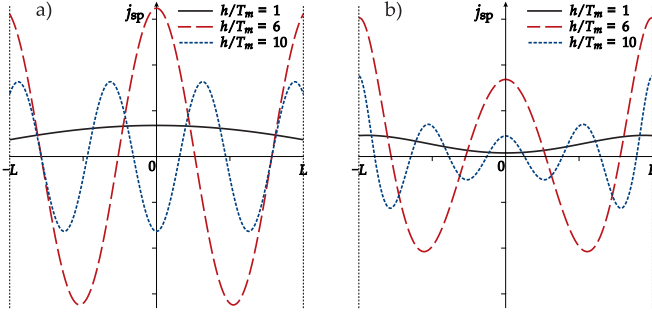


FIG. 3. (Color online.) The coordinate dependence of the spin current in the F-layer for different values of the strength of the magnetic field  $h \equiv \mathcal{H}$ ;  $T_m$  is the SDW transition temperature.

- a)  $(\mathcal{T}_{\text{eff}})^2 \propto \delta(1 - |n_x|)$ ,  
 b)  $(\mathcal{T}_{\text{eff}})^2 = \gamma_B = \text{const.}$

where

$$j_{\text{c,Sp}}^x(x) = \mu_B \nu v (2\pi T) \times \sum_{\omega} \left\langle \frac{|n_x| m^2 \mathcal{T}_{\text{eff}}^4 \tanh(\theta_{\omega}) \cos(\theta_{\mathcal{H}}) \cos(\theta_{\mathcal{H}} x/L)}{\cosh^2(\theta_{\omega}) - \cos^2(\theta_{\mathcal{H}})} \right\rangle. \quad (34)$$

One can see that the spin current oscillates in space in the F-layer. It is also seen that in the case of a normal (nonmagnetic) metal, i.e., at  $\mathcal{H} = 0$ , the spin current is constant in space.

As follows from Eq. (34), the spin current at the interfaces it is equal to

$$j_{\text{c,Sp}}^x(\pm L) = \mu_B \nu v (2\pi T) \times \sum_{\omega} \left\langle \frac{|n_x| m^2 \mathcal{T}_{\text{eff}}^4 \tanh(\theta_{\omega}) \cos^2(\theta_{\mathcal{H}})}{\cosh^2(\theta_{\omega}) - \cos^2(\theta_{\mathcal{H}})} \right\rangle. \quad (35)$$

Using Eq. (34) we plot the coordinate dependence of the critical spin current with the projection of the spin onto the  $x$ -axis in Fig. 3 for different values of  $\mathcal{H}$ . The critical spin current oscillates with a period of the order of  $\xi_{\text{Fb}}$ . On the left panel we choose  $(\mathcal{T}_{\text{eff}})^2 \propto \delta(1 - |n_x|)$  so that there is no dependence of the oscillation amplitude on  $x$ ; on the right panel we choose  $(\mathcal{T}_{\text{eff}})^2 = \gamma_B = \text{const.}$ , so that the amplitude depends on  $x$  in a power-law fashion, which is a consequence of the integration over the angles.

Thus, we see that the SDW order parameter penetrating into the ferromagnet contains two components—a long-range component with  $\mathbf{M}$  vector perpendicular to the magnetization in the ferromagnet  $\mathbf{M}_{\text{F}}$ , and a “short”-range component with  $\mathbf{M}$  vector parallel to the  $z$ -axis. A similar effect arises in an S/F/S junction with the nonhomogeneous magnetization, where the long-range component is the odd triplet ( $S \neq 0$ ) wave function of the Cooper pairs, and the short-range component corresponds to the singlet and  $S = 0$  triplet wave function.<sup>18,19</sup> However, the spin current  $j_{\text{Sp}}$  is not analogous to the charge Josephson current  $j_{\text{J}}$ —it is not constant in space, but oscillates with a period  $\sim \xi_{\text{Fb}}$ .

## IV. SDW/I/SDW “JOSEPHSON” JUNCTION

### A. Ferrell–Prange Equation

Now, we consider a junction formed by two materials with the SDW. These can be iron-based pnictides separated by an insulating, normal metal or ferromagnetic layer. In all these cases the current through the junction is given by

$$j_{\text{Sp}}^x = j_{\text{c,Sp}}^x \cdot \sin(\alpha), \quad (36)$$

where the critical spin current depends on the type of the barrier. For example, for SDW/N/SDW junction it is equal to

$$j_{\text{c,Sp}}^x = 2\mu_B \nu v (2\pi T) W_{M0}^2 \sum_{\omega} \left\langle \frac{|n_x| \mathcal{T}_{\text{eff}}^4}{\mathcal{E}_M^2 \sinh(2\theta_{\omega})} \right\rangle. \quad (37)$$

Here, the superscript  $x$  denotes the component of the spin current vector having the spin projection on the  $x$ -axis only. For the case of SDW/I/SDW junction we easily obtain

$$j_{\text{c,Sp}}^x = 2\mu_B \nu v (2\pi T) W_{M0}^2 \sum_{\omega} \left\langle \frac{|n_x| \mathcal{T}_{\text{eff}}^2}{\mathcal{E}_M^2} \right\rangle. \quad (38)$$

Introducing  $\tilde{\Gamma} = \langle |n_x| \mathcal{T}_{\text{eff}}^2 \rangle$  we can write this equation in the case of ideal nesting as

$$j_{\text{c,Sp}}^x = \tilde{\Gamma} \nu \mu_B W_{M0}^2 (2\pi T) \sum_{\omega} \mathcal{E}_M^{-2} = \frac{\mu_B W_{M0}}{e^2 \tilde{R}} \tanh \frac{W_{M0}}{2\pi T}, \quad (39)$$

which corresponds to the Ambegaokar–Baratoff formula for the critical Josephson current with  $\Delta$  replaced by  $W_{M0}$  and the “interface resistance per unit area”  $\tilde{R}$  is related to the effective transmission coefficient— $\tilde{R} \propto 1/\tilde{\Gamma}$ .

Equation (36) for the spin “Josephson” current in an AF/I/AF junction has been obtained in Ref. 6 by another method.

Now, we derive an equation similar to the Ferrell–Prange<sup>48</sup> equation. We consider a junction with SDW leads in the form of thin films. The thickness  $d$  of the films must be less than a characteristic length, over which the SDW vector  $\mathbf{M}$  restores its preferable direction. This direction is determined by anisotropy effects.

The idea of the derivation is the same as in Ref. 48. Considering the region near the boundary (cf. Fig. 4) we use the continuity equation for the spin  $\mathbf{M}^{(x)}$ ,

$$\partial_t \mathbf{M}^{(x)} = -\text{div} \mathbf{j}_{\text{Sp}}, \quad (40)$$

which can be derived easily from Eq. (3.5) of Ref. 42. Using the Green’s functions  $\check{g}$  unperturbed by the proximity effect (see Eq. (5)), one can show that  $\mathbf{M}^{(x)} = 0$ . From the continuity equation we get

$$d \cdot [\partial_z j_{\text{Sp}}^z(z)|_{x=+0} - \partial_z j_{\text{Sp}}^z(z)|_{x=-0}] = 2 \cdot j_{\text{Sp}}^x. \quad (41)$$

The formula for the spin current  $j_{\text{Sp}}^x$  in an SDW lead with the magnetization orientation varying in space can be easily

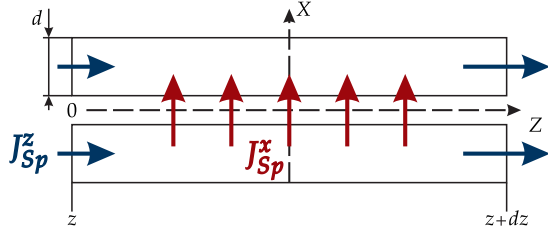


FIG. 4. (Color online.) A scheme for derivation of the Ferrell-Prange equation.

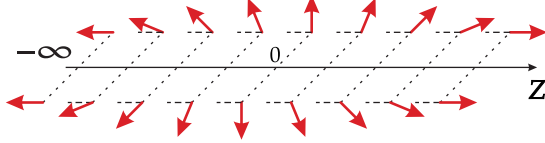


FIG. 5. (Color online.) Dependence of the magnetization direction in the “fluxon” with  $z$ . The mutual orientation in the leads changes from 0 to  $2\pi$ .

obtained from Eq. (6) in the same way as the Meissner current is found in a clean superconductor (see, e.g., Ref. 45). It has the form (see Appendix B)

$$j_{Sp}^z(z)|_{x=\pm 0} = -\frac{1}{6} \frac{\partial \alpha}{\partial z} v^2 \nu \mu_B (2\pi T) W_{M0}^2 \sum_{\omega} \mathcal{E}_M^{-3}. \quad (42)$$

This equation formally coincides with the corresponding formula for the supercurrent in a clean superconductor if one replaces  $W_{M0} \rightarrow \Delta$ ,  $\alpha \rightarrow \chi$  and  $\mu_B \rightarrow 2e$ , where  $\chi$  is the phase in the superconductor.

Substituting Eq. (36) into Eq. (41) we arrive at the final result

$$dl_c \partial_z^2 \alpha = \sin(\alpha), \quad (43)$$

where  $l_c$  is a characteristic length inversely proportional to the critical current  $j_{c,Sp}$ . It equals

$$l_c = \left( \sum_{\omega} \left\langle \frac{|n_x| \mathcal{T}_{\text{eff}}^4}{\mathcal{E}_M^2 \sinh(2\theta_{\omega})} \right\rangle \right)^{-1} \frac{\hbar v}{3} \sum_{\omega} \mathcal{E}_M^{-3} \quad (44)$$

in the case of SDW/N/SDW junctions and

$$l_c = \left( \sum_{\omega} \left\langle \frac{|n_x| \mathcal{T}_{\text{eff}}^2}{\mathcal{E}_M^2} \right\rangle \right)^{-1} \frac{\hbar v}{3} \sum_{\omega} \mathcal{E}_M^{-3} \quad (45)$$

for SDW/I/SDW junctions.

Equation (43) is the analog of the Ferrell-Prange equation for a junction composed of two materials with the SDW and separated by some barrier (a thin insulating or normal metal layer).

One can easily find a solution similar to the one describing a fluxon in the Josephson tunnel junctions. It has the form

$$\alpha = 4 \arctan [\exp(z/l_0)], \quad (46)$$

where the size of the “fluxon” is given by  $l_0 = \sqrt{dl_c}$ . The mutual orientation changes along the  $z$ -axis from 0 to  $2\pi$  and has the value  $\pi$  in the core of the “fluxon”, cf. Fig. 5. In the case of the Josephson fluxon one has a circulating charge current—in our case, the spin current circulates in the “fluxon”.

## B. Finite Detuning

Up to now, we assumed the ideal nesting,  $\delta\mu = 0$ . In the case  $\delta\mu \neq 0$  we find the quasiclassical Green's function in the SDW system by inverting the matrix  $\check{G}$  in Eq. (3.3) of Ref. 42. It turns out that  $\check{g}$  is a linear combination of six “basis matrices”

$$\check{g}_{L/R} = \check{g}_{030} \pm \check{g}_{122} + \check{g}_{123} \pm \check{g}_{212} + \check{g}_{213} + \check{g}_{300}, \quad (47)$$

where the subscripts L and R denote the left and right SDW systems, respectively, corresponding to the setup depicted in Fig. 1 with the F-layer replaced by a very thin insulating barrier. The six functions  $\check{g}_{ijk}$  are

$$\check{g}_{030} = |\chi_+|^{-2} (\omega \cdot \Re \{\chi_+\} + \delta\mu \cdot \Im \{\chi_+\}) \cdot \hat{\rho}_0 \cdot \hat{\tau}_3 \cdot \hat{\sigma}_0, \quad (48)$$

$$\check{g}_{122} = i|\chi_+|^{-2} W_{M0} \cdot \Im \{\chi_+\} \sin(\alpha/2) \cdot \hat{\rho}_1 \cdot \hat{\tau}_2 \cdot \hat{\sigma}_2, \quad (49)$$

$$\check{g}_{123} = |\chi_+|^{-2} W_{M0} \cdot \Re \{\chi_+\} \cos(\alpha/2) \cdot \hat{\rho}_1 \cdot \hat{\tau}_2 \cdot \hat{\sigma}_3, \quad (50)$$

$$\check{g}_{212} = |\chi_+|^{-2} W_{M0} \cdot \Re \{\chi_+\} \sin(\alpha/2) \cdot \hat{\rho}_2 \cdot \hat{\tau}_1 \cdot \hat{\sigma}_2, \quad (51)$$

$$\check{g}_{213} = i|\chi_+|^{-2} W_{M0} \cdot \Im \{\chi_+\} \cos(\alpha/2) \cdot \hat{\rho}_2 \cdot \hat{\tau}_1 \cdot \hat{\sigma}_3, \quad (52)$$

$$\check{g}_{300} = i|\chi_+|^{-2} (\omega \cdot \Im \{\chi_+\} - \delta\mu \cdot \Re \{\chi_+\}) \cdot \hat{\rho}_3 \cdot \hat{\tau}_0 \cdot \hat{\sigma}_0, \quad (53)$$

where  $\chi_+ = \sqrt{W_{M0}^2 + (\omega + i\delta\mu)^2}$ .

The detuning parameter  $\delta\mu$  can be written in the form  $\delta\mu = \mu_0 + \mu_\phi \cos(2\phi)$  with  $\mu_0$  describing the relative size mismatch of the “hole” and “electron” pockets and  $\mu_\phi$  describes the ellipticity of the “electron” pocket, while the “hole” pocket is assumed to be a circle.<sup>31,33,34</sup>

From the quasiclassical Green's function, Eq. (47), we can calculate the dependence of various quantities of interest on  $\delta\mu$ .

We begin with the density of states (DoS). The DoS is given by the formula

$$\nu(\epsilon) = \frac{1}{2} \Re \langle \text{Tr}(\hat{\rho}_0 \cdot \hat{\tau}_3 \cdot \hat{\sigma}_0 \cdot \check{g}(\epsilon)) \rangle, \quad (54)$$

where the angle brackets mean the averaging over the momentum directions

$$\langle (\dots) \rangle = \int \frac{d\Omega}{4\pi} (\dots). \quad (55)$$

Inserting the expression for the quasiclassical Green's function into Eq. (54) we obtain

$$\nu(\epsilon) = \frac{1}{2} \Re \left\langle \frac{\epsilon + \delta\mu}{\sqrt{(\epsilon + \delta\mu)^2 - W_{M0}^2}} + \frac{\epsilon - \delta\mu}{\sqrt{(\epsilon - \delta\mu)^2 + W_{M0}^2}} \right\rangle. \quad (56)$$

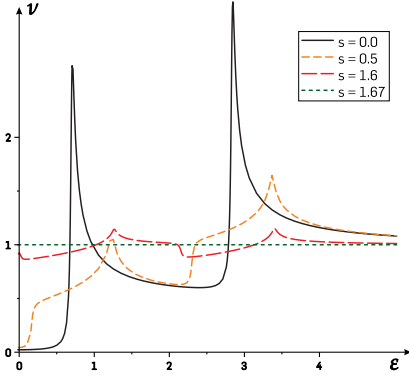


FIG. 6. (Color online.) DoS for different values of  $s = \mu_\phi/\mu_0$  at  $\mu_0/T_s \approx 1.07$ .

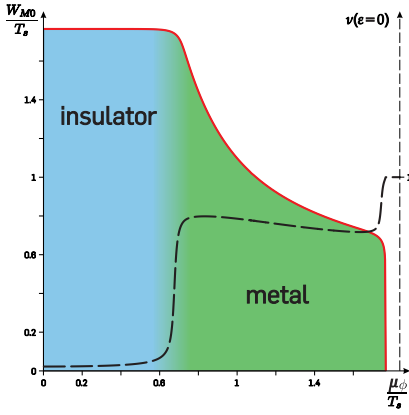


FIG. 7. (Color online.)  $W_{M0}(\mu_\phi)$  at  $\mu_0/T_s \approx 1.07$ ; black dashed curve shows the dependence of DoS on  $\mu_\phi$  at zero energy.

This result was obtained earlier in Refs. 33 and 34.

We plot the DoS for different values of the ratio  $s = \mu_\phi/\mu_0$  in Fig. 6 choosing a constant  $\mu_0 \approx 1.07T_s$ , where  $T_s$  is the SDW order transition temperature. Note that there is a range of the values of  $s$  where an energy gap appears in the DoS, indicating an insulating phase of the system. We can trace the transition to the metal considering the value of the DoS at zero energy,  $\nu(0)$ , as a function of the parameter  $\mu_\phi$ . In Fig. 7 we plot the dependence of the order parameter  $W_{M0}$  on  $\mu_\phi$  and, in the same plot, show the dependence  $\nu(0)$  on  $\mu_\phi$  (dashed line). One can observe a rather steep increase of  $\nu(0)$  in the region around  $0.7T_s$ .

The dependence of the SDW order parameter  $W_{M0}$  on  $\mu_\phi$  is calculated from the self-consistency equation, Eq. (3.19) of Ref. 42, which in addition has to be averaged over the momentum directions. It reduces to the form (see also Ref. 33)

$$\ln(T/T_s) = 2\pi T \sum_{\omega>0} \Re \langle \chi_+^{-1} - \omega^{-1} \rangle. \quad (57)$$

It is of special interest<sup>3</sup> to calculate the spin current through the SDW/I/SDW system. Using the expression Eq. (47) for the quasiclassical Green's functions in the left

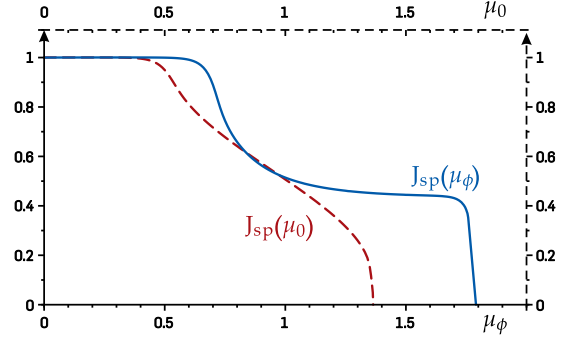


FIG. 8. (Color online.) Spin current on  $\mu_0$  resp.  $\mu_\phi$ .

and right leads, and inserting it into the expression for the spin current, Eq. (3.13) of Ref. 42, we obtain the familiar form of the Josephson-like current

$$j_{sp} = j_{c,sp} \cdot \sin(\alpha) \quad (58)$$

for the spin current through the SDW/SDW interface which has only the spin projection on the  $x$ -axis. The critical coefficient  $j_{c,sp}$  is dependent on  $\delta\mu$

$$j_{c,sp} = 2\mu_B \nu W_{M0}^2 (2\pi T) \left\langle |n_x| \mathcal{T}_{\text{eff}}^2 \sum_{\omega>0} \frac{\Re \{\chi_+\}^2 - \Im \{\chi_+\}^2}{|\chi_+|^4} \right\rangle. \quad (59)$$

In Fig. 8 we show the dependence of the critical spin current on the two parameters  $\mu_0$  and  $\mu_\phi$ , choosing in each case the other parameter constant—when plotted as a function of  $\mu_0$ , we set  $\mu_\phi = 1.26T_s$ ; plotting the spin current as a function of  $\mu_\phi$  we set  $\mu_0 = 1.07T_s$ .

One can see from Fig. 8 and Fig. 7 that in the insulating phase the spin current is not zero. In contrary, it is at its maximum. This addresses the question about the possibility of the appearance of the spin current in an insulator (see discussion on p. 12 of Ref. 3).

Analyzing the expression (59) for the critical spin current we see that there is a possibility of a sign change. At small temperatures, the spin current changes sign if, e.g., the condition  $W_{M0} \lesssim \mu_0 - \mu_\phi$  holds with  $\mu_\phi < \mu_0$  at the same time.

We note the similarity of the self-consistency equation (57) with the self-consistency equations presented in Refs. 49 and 50. Setting  $\mu_\phi = 0$  we can map the problem of finding the dependence of  $W_{M0}$  on  $\mu_0$  onto the problem of finding the dependence of the superconducting order parameter on a strong exchange field resulting in the so-called Larkin-Ovchinnikov-Fulde-Ferrell (LOFF) state, which is characterized by the spatial dependence of  $\Delta$ . Note that the possibility of the LOFF-like state in pnictides was noted earlier by Gor'kov and Teitelbaum.<sup>55</sup>

## V. DISCUSSION

We have considered Josephson-like junctions of the SDW/F/SDW and SDW/I/SDW types and calculated the



spin current  $j_{\text{sp}}$  in these systems. In both the cases the dependence of  $j_{\text{sp}}$  on the angle  $\alpha$  between the mutual orientations of the magnetization vectors  $\mathbf{M}$  in the right and left leads is given by Eq. (36). The critical current density  $j_{\text{sp,c}}$  is determined by the interface transparencies.

We emphasize the analogy between the SDW/F/SDW junctions and the Josephson S/F/S junctions with a nonhomogeneous magnetization. In both the systems there are “short”- and long-range components of the order parameters that penetrate the ferromagnet. In the ballistic systems under consideration the “short”-range SDW component penetrating the F-layer oscillates with a period of the order of  $\xi_{\text{fb}} = \hbar v / \mathcal{H}$ . It stems from the SDW component parallel to the magnetization vector  $\mathbf{M}_F$  in the ferromagnet. The SDW component normal to the  $\mathbf{M}_F$  vector penetrates the ferromagnet on a long distance of the order of  $\xi_{\text{N,b}} = \hbar v / 2\pi T$ . The penetration depth of this component does not depend on the exchange field  $\mathcal{H}$ , and, in this sense, this component is analogous to the odd triplet long-range component in S/F/S junctions with a nonhomogeneous magnetization.

However, there is an essential difference between the spin current in SDW/F/SDW junctions  $j_{\text{sp}}(x)$  and the Josephson charge current  $j_j$  in S/F/S junctions. Whereas the Josephson current  $j_j$ , as it should be, does not depend on  $x$  ( $\text{div } j_j = \partial_x j_j = 0$ ), the spin current  $j_{\text{sp}}(x)$  oscillates in space with a period of the order  $\xi_{\text{fb}}$ . This behavior is a consequence of the fact that the spin current arises due to the interference between the “short”- and long-range component of the SDW in the ferromagnet.

The spin current in SDW/I/SDW junctions depends on the angle  $\alpha$  in the same way as in the SDW/F/SDW junctions, i.e.,  $j_{\text{sp}} \sim \sin(\alpha)$ . We have calculated the dependence of  $j_{\text{sp}}$  in an SDW/I/SDW junction on the parameter  $\delta\mu$  characterizing the deviation from the ideal nesting. At  $\delta\mu = 0$ , an energy gap  $W_{M0}$  opens in the excitation spectrum in the leads with the SDW (see Fig. 7). In a certain region of the parameter  $\delta\mu$ , the energy gap disappears and the leads of the junction become metals (or semimetals). The spin current exists in both the cases; it turns to zero at the Neel temperature when  $W_{M0} \rightarrow 0$ .

Furthermore, we have derived the Ferrell–Prange equation for the Josephson-like junctions with the SDW and found a solution describing a localized spin current distribution analogously to the fluxon in a tunnel S/I/S Josephson junction. The characteristic length of the “fluxon”,  $l_0 = \sqrt{d\mathcal{T}_c}$ , is determined by the barrier transmittance and the thickness  $d$  of the leads with the SDW. In the case of bulk leads the thickness  $d$  should be replaced by a characteristic length over which the vector  $\mathbf{M}$  of the SDW restores its favorable direction in the bulk, which is determined by the anisotropy effects.

#### A. Acknowledgements

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#### Appendix A: Model Boundary Condition

Here, we generalize the boundary conditions for quasiclassical Green’s functions to the case of a two-band material with the SDW. It is not in the scope of the paper to present a rigorous derivation of the boundary conditions with account for all the possible processes accompanying the passage of electrons through the interface. We ignore the interband transitions and also the spin-flip processes. In addition, we restrict ourselves to the case of a low transmittance coefficient. Thus, the derived boundary conditions, to some extent, are phenomenological. However, in a single-band case they coincide with the boundary conditions derived by Zaitsev<sup>46</sup> and in case of two-band material they are compatible with the boundary conditions used in Ref. 53. Note also that the exact form of the boundary conditions is not important for our qualitative conclusions such as the appearance of the “short”- and long-range SDW components in the ferromagnet.

In the case of weak penetration and single-band materials, the boundary conditions for the Green’s function  $\check{g}$  have the form of Eq. (7), in which the matrix  $\check{\mathcal{T}}$  should be replaced by a scalar  $\mathcal{T}$  (see Ref. 46). The Green’s function on the left-hand side of this equation determines the charge current through the interface. It equals the commutator multiplied by  $\mathcal{T}^2$  and consisting of the symmetric in momentum space Green’s functions. This part of Eq. (7), which of course also determines the charge current, can be obtained by using the tunneling Hamiltonian method (see, e.g., Ref. 54). In terms of the operators  $\check{C}_{l(r)}$  and  $\check{C}_{l(r)}^\dagger$  introduced in Ref. 42 the tunneling Hamiltonian can be written as follows

$$\mathcal{H}_T = \sum_{\mathbf{k}, s, s'} \left( \check{\mathcal{T}} \check{C}_l^\dagger \check{C}_r + \text{c.c.} \right), \quad (\text{A1})$$

where  $\check{\mathcal{T}} = (\mathcal{T}_3 \hat{\rho}_3 + \mathcal{T}_0 \hat{\rho}_0) \cdot \hat{\tau}_3 \cdot \hat{\sigma}_0$  and the coefficients  $\mathcal{T}_{0,3}$  are defined in Eq. (7) in terms of the matrix elements  $T_{1,2}$  describing the electron transition through the interface in each band. The Green’s function  $\hat{G}$  satisfies the equation (see, e.g., Ref. 54)

$$\hat{G}_l = \hat{G}_{0,l} + \hat{G}_{0,l} \cdot \hat{\Sigma}_T \cdot \hat{G}_l, \quad (\text{A2})$$

where  $\hat{\Sigma}_T = \check{\mathcal{T}} \hat{G}_r \check{\mathcal{T}}$ . This means that the self-energy part has formally the same form as in the case of the impurity scattering, but the Green’s function in  $\hat{\Sigma}_T$  describes another (right) lead in the equation for the Green’s function  $\hat{G}_l$  of the left lead. The charge current through the interface is determined by the trace of a commutator

$$\text{Tr} \{ \hat{\rho}_3 \cdot \hat{\tau}_3 \cdot \hat{\sigma}_0 \cdot [\hat{\Sigma}_T, \hat{G}_l] \} = \text{Tr} \{ \hat{\rho}_3 \cdot \hat{\tau}_3 \cdot \hat{\sigma}_0 \cdot [\check{\mathcal{T}} \hat{G}_r \check{\mathcal{T}}, \hat{G}_l] \}. \quad (\text{A3})$$

The expression in the curly brackets expressed in terms of the quasiclassical Green’s functions appears in the right-hand side of Eq. (7).

## Appendix B: Spin Current in a Bulk Material with the SDW

In order to obtain the expression for the spin current in the left (right) lead flowing parallel to the interface, we assume that the angle  $\alpha$  varies in space slowly so that the change of  $\alpha$  on a short coherence length  $\xi_{\text{SDW}} = \hbar v / W_{M0}$  is small. The characteristic length  $\xi_\alpha$  of the  $\alpha$  variation is inversely proportional to the critical current  $j_{c,\text{Sp}}^x$  (see below) and is much larger than  $\xi_{\text{SDW}}$ . If the angle  $\alpha$  is spatially constant, then the Green's function  $\check{g} \equiv \check{g}_\alpha$  obeys the equation

$$[\check{A}, \check{g}_\alpha] = 0. \quad (\text{B1})$$

The solution of this equation is given by Eq. (5) and may be represented in the form

$$\check{g}_\alpha = \check{A} / \mathcal{E}_M = \check{S} \check{g}_0 \check{S}^\dagger, \quad (\text{B2})$$

where the matrix  $\check{g}_0 = \{\omega_n \hat{\tau}_3 + (W_{M0} / \mathcal{E}_M) \hat{\rho}_1 \cdot \hat{\tau}_2 \cdot \hat{\sigma}_3\}$  does not depend on  $\alpha$ . The unitary matrix  $\check{S}$  is defined as  $\check{S} = \cos(\alpha/2) + i \sin(\alpha/2) \hat{\rho}_3 \cdot \hat{\tau}_3 \cdot \hat{\sigma}_1$ . If the angle  $\alpha(z)$  slowly

depends on the coordinate along the interface, we have to add a correction  $\delta \check{g}_\alpha$  to the matrix  $\check{g}_\alpha$  which depends on  $n_x \rightarrow \delta \check{g}_\alpha = n_x \check{a}_\alpha$ . The matrix  $\check{a}$  satisfies the equation

$$\nu(\check{S}^\dagger \cdot \partial_z \check{S} \cdot \check{g}_0 + \check{g}_0 \cdot \partial_z \check{S}^\dagger \cdot \check{S}) + 2\mathcal{E}_M \check{g}_0 \cdot \check{a}_0 = 0, \quad (\text{B3})$$

where the matrices  $\check{a}_\alpha$  and  $\check{a}_0$  are related in the same way as matrices  $\check{g}_0$  and  $\check{g}_\alpha$ , i.e.,  $\check{a}_\alpha = \check{S} \check{a}_0 \check{S}^\dagger$ . Here, we used

$$\check{g}_0 \cdot \check{a}_0 + \check{a}_0 \cdot \check{g}_0 = 0, \quad (\text{B4})$$

which follows from the normalization condition Eq. (27). One easily finds

$$\check{S}^\dagger \partial_z \check{S} = (i/2) \partial_z \alpha \hat{\rho}_3 \cdot \hat{\tau}_3 \cdot \hat{\sigma}_1. \quad (\text{B5})$$

Combining Eqs. (B3), (B5) and the normalization condition  $\check{g}_0 \cdot \check{g}_0 = 1$ , we find

$$\check{a}_0 = -\nu(i/2) \partial_z \alpha W_{M0} \mathcal{E}_M^{-3} (-i\omega \hat{\rho}_0 \cdot \hat{\tau}_3 \cdot \hat{\sigma}_2 + W_{M0} \hat{\rho}_3 \cdot \hat{\tau}_3 \cdot \hat{\sigma}_1), \quad (\text{B6})$$

and using Eq. (32), we obtain the Eq. (42) for  $j_{\text{Sp}}^z(z)$ .

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